Use of the ρ-Photon Analogy in a Model of Isobar Production*†

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A model for the production of isobars by the exchange of a spin-1 meson is presented. In particular we discuss the production of the $N^*(1238)$ and $Y^*(1380)$ $P_{3/2}$ isobars by π, K^+ and K^- mesons by the exchange of ρ or K^*. The vertex $\rho+N \to N+\pi$ or $\bar{K}^*+N \to \Lambda+\pi$ is treated by assuming a magnetic dipole transition ($M1 \to P_{3/2}$) in analogy with virtual photoproduction (electroproduction). This assumption then leads to a prediction for the decay distribution, the over-all production distribution and the mass distribution of the isobar with no free parameters. This description is found to be in agreement with experiment in a variety of cases, although to obtain quantitative agreement with the production angular distributions at higher energies a form factor must be assumed to give sufficient backward peaking for the isobar. It is further shown how the absolute cross section for isobar production by this mechanism may be found in terms of the photoproduction cross section by assuming the "ρ-photon analogy" (i.e., total ρ dominance of isovector photon interactions) which at zero momentum transfer reads

$$(1/e)\langle J_\mu{}^{\gamma v}\rangle = (1/f_\rho)\langle J_\mu{}^\rho\rangle.$$

The resulting number is reasonable as far as order of magnitude is concerned. To make a quantitative comparison with experimental cross sections, the reaction $K^+ + P \to N^{*++} + K^0$, where some detailed information exists, is examined. The value of the ρKK coupling needed here is found in terms of the known $\rho\pi\pi$ coupling by assuming universal coupling of the ρ to the isospin current. The resulting theoretical cross section is found to be too small by roughly a factor of six at 910 MeV/c and in rough agreement within theoretical uncertainties at 1.4 BeV/c and 1.96 BeV/c if the effect of the form factor in reducing the cross section is taken into account. Formulas for the decay of an arbitrary isobar excited by spin-1 exchange and a test for spin-1 exchange are given. In the Appendices, production of ω^0 with N^* is discussed and a brief treatment in terms of N^* as a spin-$\frac{3}{2}$ particle is given.

I. INTRODUCTION

THE ρ meson, or the 780 MeV, $T=1$, $J^P=1^-$, $\pi-\pi$ resonance, plays the central role in this model even though it never "appears." The existence of such a resonance, although with a somewhat lower mass, was suggested by Frazer and Fulco[1] in connection with their study of the electromagnetic form factors of the nucleon. Later, Sakurai,[2] associating it with the Yang–Mills field,[3] suggested its existence could be motivated by considering it as one of three spin-1 mesons, each one universally coupled to a current conserved within the realm of strong interactions, an analogy with the photon's universal coupling to the electric current—in this case the ρ being coupled to the isotopic spin current. This implies the effective Lagrangian density

$$\mathcal{L} = -f_\rho[i\bar{\psi}_N\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}\psi + \boldsymbol{\phi}_\pi\times(\partial_\mu\boldsymbol{\phi}_\pi) + i(\partial_\mu\phi_K{}^+)\tfrac{1}{2}\boldsymbol{\tau}\phi_K - i\phi_K{}^+\tfrac{1}{2}\boldsymbol{\tau}(\partial_\mu\phi_K) + \cdots]\cdot\boldsymbol{\rho}_\mu. \quad (1)$$

Subsequently, this resonance was actually "seen"[4] in the mass plots of high-energy pion production reactions and its quantum numbers were verified. From the width of the observed resonance (≈ 100 MeV) the coupling of the ρ to two π's $f(\rho\pi\pi)$, could be estimated, thereby opening the way for a check on the universality of the coupling to the isotopic spin current.[5] While the Frazer–Fulco analysis could be loosely characterized by saying the ρ mediates the interaction between the (isovector part of the) photon and the nucleon, Gell-Mann and Zachariasen[6] exploiting universality, crystallized what we shall call the "ρ-photon analogy" by saying: "In *all* problems each matrix element for a virtual isovector γ ray (to lowest order in e) can be expressed in terms of the corresponding matrix element for a virtual ρ meson by multiplying by the factor $(e/f_\rho)(-m_\rho^2/s-m_\rho^2)$," [where we have used a form of the coupling constants corresponding to Eq. (1)]. A similar conjecture, of course, is also made for the isoscalar interactions of the photon with strongly interacting particles. In this paper, Gell-Mann and Zachariasen also suggested how the usual techniques of making calculations with vector mesons in terms of neglecting their instability and using Feynman diagrams could be put on a somewhat more satisfactory footing. Finally, unitary symmetry[7] suggests that we place the ρ in a octet of similarly interacting vector mesons[8]—consisting of the ρ, K^*, $\bar{K}^*$, and a neutral vector meson, presumably a linear combination of the ω and ϕ mesons.

In view of the intrinsic theoretical interest and elegance in the idea of the ρ as a "heavy (isovector)

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[1] W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).
[2] J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).
[3] C. N. Yang and R. L. Mills, Phys. Rev. 98, 1501 (1954).
[4] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters 6, 628 (1961).
[5] J. J. Sakurai, Proceedings of the International School of Physics "Enrico Fermi," Varenna, Italy (unpublished).
[6] M. Gell-Mann and F. Zachariasen, Phys. Rev. 134, 953 (1961).
[7] M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
[8] J. J. Sakurai, in *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), p. 227.

photon" it would seem desirable to find situations in which quite specific aspects of the analogy may be exploited and tested. Production of the (3,3) resonance, or $N^*(1238)$ isobar, via one ρ exchange, by π or K mesons on nucleons presents an interesting opporunity. In this situation one π exchange is forbidden, so that it might be possible to isolate the ρ exchange effect.

The ρ vertex involved would then be of the type

$$\rho+N \rightarrow N^* \rightarrow N+\pi\,,$$

which we then can view in terms of the ρ-photon analogy as "photoproduction" off the mass shell—a process closely related to $\gamma+N \rightarrow N+\pi$, which has been well studied experimentally, particularly with $N\pi$ energies in the resonance region. Note that since only the isovector part of the photon enters in exciting the

$$T=\tfrac{1}{2},\, N \quad \text{to a } T=\tfrac{3}{2},\, N^*,$$

the real-life photoproduction is entirely parallel to our hypothesized "strong photoproduction." The electromagnetic analog of our problem then is the electroproduction of pions:

$$e^- + p \rightarrow e^- + N + \pi\,.$$

This reaction was discussed by Dalitz and Yennie.[9] Later Fubini, Nambu, and Wataghin,[10] and others[11] used dispersion relations, particularly to discuss the effects of high-momentum transfers. Experiments on electroproduction have confirmed the general expectations of the theoretical treatments. The essential idea used in the theory of electroproduction is that the field of the fast moving electron appears as a cloud of "almost real" photons one of which "strikes" the proton, creating a pion by "photoproduction." In field-theoretic terminology, the pion is produced by one-photon exchange. The effects of more photon exchanges are neglected because a factor of e must enter with each photon, thus strongly reducing the effect of such higher exchanges. In our case, since all our particles are strongly interacting, we cannot so easily discount the effects of other mechanisms. We merely appeal to the not yet fully understood pragmatic success of one-particle exchange models,[12] and hope that situations can be found where the mechanism can be isolated. As we shall see, this seems to be possible. The diagram corresponding to the mechanism is shown in Fig. 1. The parts played by the initial and final electron are now taken by the initial and final "peripheral" mesons labeled q_1 and q_2, the photon has become a ρ, while the proton and "extra" pion q remain as in electroproduction. The algebra of

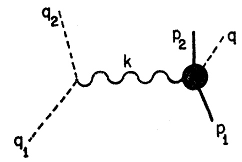


FIG. 1. Diagram for isobar production by vector boson exchange.

this problem is actually simpler than that of electroproduction. In the latter case the interaction is taken to be $J_\mu \mathcal{E}_\mu$. J_μ is the matrix element of the electromagnetic current connecting N to $N\pi$ states while $\mathcal{E}_\mu$ is $e\bar{u}(q_2)\gamma_\mu u(q_1)$, the Möller potential of the electron. But in our case $\mathcal{E}_\mu$ is essentially just $f(\rho\pi\pi)(q_1+q_2)_\mu$, which is much simpler due to the absence of spin.

If we use an idea like the octet model which states that the K^* ($J^P=1^-$, 880 MeV, πK resonance) interacts similarly to the ρ and that the Y_1^* ($P_{3/2}$, 1380 MeV $\Lambda\pi$ resonance) interacts similarly to the N^*, then we can extend our chain of association to include reactions like $K^-+P \rightarrow Y^*+\pi$ and $\pi^++P \rightarrow Y^*+K$, where a $\bar{K}^*$ is exchanged and our vector meson–nucleon-isobar vertex becomes $\bar{K}^*+P \rightarrow Y^* \rightarrow \Lambda+\pi$.

For convenience we summarize our notation here. The scalar product of two four-vectors is $A\cdot B=\mathbf{A}\cdot\mathbf{B}+A_4B_4=\mathbf{A}\cdot\mathbf{B}-A_0B_0$, and q_1 is the four-momentum of incident meson; q_2 is the four-momentum of final "peripheral" meson; q is the four-momentum of "extra" meson produced as part of isobar; p_1 is the four-momentum of incident nucleon; p_2 is the four-momentum of final nucleon; $Q=q+p_2=$ four-momentum of isobar; $-Q^2=M^{*2}=$ (mass)^2 of isobar; $K=q_1-q_2$, four-momentum of virtual ρ; $t=-K^2=-(q_1-q_2)^2$, square of invariant momentum transfer; $s=-(p_1+q_1)^2=-(Q+q_2)^2$, total c.m. (energy)^2; (*) means "as evaluated in the rest frame of the isobar."

A letter by Sakurai and the author[13] presented some preliminary results of this model. The essential idea was: Since the photoproduction matrix element for $\gamma+P \rightarrow N^* \rightarrow P+\pi$ is known to correspond to a magnetic dipole transition ($M1 \rightarrow P_{3/2}$), and since the electroproduction investigations have indicated this does not change essentially as we move t away from zero, then the vertex $\rho+N \rightarrow N^* \rightarrow N+\pi$ must also go via $M1 \rightarrow P_{3/2}$. This means that in the isobar rest frame we have the matrix element $M=(3\mathbf{q}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{K}\times\boldsymbol{\varepsilon})$ by assumption, where $\boldsymbol{\varepsilon}$, however, the space part of the polarization of the virtual ρ, is $(\mathbf{q}_1+\mathbf{q}_2)^*$. This then means that since $(\mathbf{K}\times\boldsymbol{\varepsilon})^*=2(\mathbf{q}_1\times\mathbf{q}_2)^*$, the decay distribution of the isobar is $[1+3(\hat{q}\cdot\hat{n})^2]$, where $\mathbf{n}$ is the normal to the production plane, $\mathbf{n}=\mathbf{q}_1\times\mathbf{q}_2$. Furthermore, since $(\mathbf{q}_1\times\mathbf{q}_2)^*$ varies with the production angle $\theta_{\text{c.m.}}$ in the over-all c.m. ($\mathbf{q}_1\cdot\mathbf{q}_2=\cos\theta_{\text{c.m.}}$), then the distribution in $\theta_{\text{c.m.}}$ for the "two-body" process $\pi=N \rightarrow N^*+\pi$ varies as $\sin^2\theta_{\text{c.m.}}/(t-m_\rho^2)^2$.

As this work was being concluded, it was brought to

[9] R. H. Dalitz and D. R. Yennie, Phys. Rev. **105**, 1598 (1957).

[10] S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. **111**, 329 (1958).

[11] I. M. Barbour, Nuovo Cimento **27**, 1382 (1963). This paper contains further references to experimental and theoretical work on electroproduction. See also Ph. Salin, University of Bordeaux (to be published).

[12] E. Ferrari and F. Selleri, Nuovo Cimento Suppl. **24**, 453 (1962), give a list of references on the one-pion exchange model.

[13] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters **11**, 90 (1963).

our attention that Solov'ev and Ch'en Ts'ung-Mo[14] have noted on similar grounds that this angular distribution should follow in $\pi+N\to N^*+\pi$. These authors also suggested a method for calculating the size of the cross section for this reaction related to, but different from that which we present in the next section. In Sec. III we compare the predictions of the model with experiments for several reactions.

II. ABSOLUTE CROSS SECTION

Thus far we have exploited the hypothesis that the ρ meson and the isovector component of the photon are coupled to essentially the same current only to the extent that we have taken the matrix elements for the ρNN^* and γNN^* couplings to have the same algebraic form, namely, $3\mathbf{q}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}\;\boldsymbol{\sigma}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}$; however, we can use the idea that the ρ and isophoton currents are actually proportional to attempt to estimate the absolute cross section for isobar production, thereby perhaps presenting more stringent tests of the concepts we are using.

We can proceed as follows: To calculate the cross section we need to know the absolute strengths of the two vertices of our diagram (Fig. 1) where the ρ meson is coupled. On the side where the ρ is coupled to the two pseudoscalar bosons, we can use the value of $f^2(\rho\pi\pi)/4\pi \simeq 2.2$ given by the experimental width of the ρ.[5] If the bosons are not π's we may use some symmetry assumption, such as that implied by Eq. (1), to relate the relevant coupling constants to $f(\rho\pi\pi)$. If a K^* is exchanged, one may again use the width of the K^* to give $f(K^*\pi K)$. This involves an extrapolation of the coupling constants from the mass of the vector boson to negative values of t, but we presume the form factors are not varying drastically. At the other side of the diagram, we must determine the strength of the ρNN^* coupling. However, we know the magnitude of the γNN^* interaction since that is merely photoproduction. If we now use our assumption that the ρ dominates isovector photon interactions, then the photoproduction process may be diagrammed[15] as in Fig. 2. If we know the ρ-photon coupling we may then "divide out" the photon in Fig. 2 and relate our results directly to the photoproduction cross section. This effective ρ-photon coupling in fact must be related[5,7] at zero-momentum transfer to the ρNN coupling constant by $\gamma_{\gamma\rho}=em_\rho^2/f(\rho NN)$. Again we will leave aside for the moment complications due to

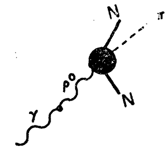


FIG. 2. Effective diagram for photoproduction into $T=\frac{3}{2}$ state, assuming ρ dominance of isovector photon interactions.

the extrapolation of the photoproduction process to photons of negative (mass)^2. Specifically, we write the isovector electromagnetic current connecting a proton and a proton-π^0 state as

$$\langle P\pi^0|J_\mu^{\gamma_v}|P\rangle=\bar{u}(P_2)M_\mu u(P_1)\,, \tag{2}$$

where $\bar{u}$ and u are the initial and final proton spinors and M_μ is a four-vector constructed from γ matrices and the energy-momentum four-vectors of the particles involved. We can express the photoproduction cross section in terms of this:

$$T=(2\pi)^4\delta^4(P_1+K-P_2-q)\times(M_N^2/P_{01}P_{02}2q_0 2K_0)^{1/2}\times\bar{u}(P_2)M_\mu u(P_1)\,\mathcal{E}_\mu\,,$$

$$\sigma(\gamma P\to P\pi^0)=\frac{1}{16\pi^2}\left(\frac{M_N}{M^*}\right)^2\frac{|\mathbf{q}^*|}{K_\gamma}\int d\Omega_{\mathbf{q}}\times\sum|\bar{u}(P_2)M_\mu u(P_1)|^2\,, \tag{3}$$

where M^*= total mass of the system, and the (*) on the photon and pion momenta are for later convenience, indicating evaluation at the energy of the $P\pi$ system.

Now if we consider the process $\pi^++P\to\pi^++P+\pi^0$ going via ρ^0 exchange, the $\rho^0+P\to P+\pi^0$ vertex is analogous to the photoproduction process and the T matrix element corresponding to Fig. 1 is

$$T=(2\pi)^4\delta^4(P_1+q_1-P_2-q_2-q)\left(\frac{M_N^2}{P_{10}P_{20}2q_{20}2q_{10}2q_0}\right)^{1/2}\times f(\rho\pi\pi)(q_1+q_2)_\mu\frac{\delta_{\mu\nu}-K_\mu K_\nu/m_\rho^2}{t-m_\rho^2}\langle P\pi^0|J_\nu^{\rho^0}|P\rangle \tag{4}$$

[where $t=-(q_1+q_2)^2$].

The ρ-photon analogy,

$$\langle P\pi^0|J_\nu^{\rho^0}|P\rangle_{t=0}=(f(\rho NN)/e)\langle P\pi^0|J_\nu|P\rangle_{t=0}=(f(\rho NN)/e)\bar{u}(P_2)M_\mu u(P_1)\,,$$

will allow us to relate our results to $\sigma(\gamma P\to P\pi^0)$. By introducing the four-vectors $\frac{1}{2}(q-P_2)$ and P_2+q so that P_2+q is the four-momentum vector of the $P\pi^0$ system as a whole, $-(P_2+q)^2=M^{*2}$, and $\frac{1}{2}(q-P_2)\to\mathbf{q}^*$, the momentum of the π^0 in the $P\pi^0$ center of mass, the phase-space integrations for our three-body final state

[14] L. D. Solov'ev and Ch'en Ts'ung-Mo, Zh. Eksperim. i Teor. Fiz. **42**, 526 (1962) [English transl.: Soviet Phys.—JETP **15**, 369 (1962)], Sec. 8.

[15] The effective interaction corresponding to this diagram, $\rho_\mu{}^0A_\mu$, may at first sight appear to violate gauge invariance, since under $A_\mu\to A_\mu+\partial_\mu\Lambda$ we get an extra term $\rho_\mu{}^0(\partial_\mu\Lambda)$; however, since we are assuming that the ρ couples to conserved currents, we can show $\partial_\mu\rho_\mu=0$. Therefore, using an integration by parts, we can apply the ∂_μ to the $\rho_\mu{}^0$ and the extra term vanishes. On the other hand, the more sophisticated approach of Ref. 6, which also motivates the assumption that the ρ can dominate the isovector photon interactions, gives a result equivalent to this without explicitly introducing a $\rho-\gamma$ coupling.

can be broken up so that

$$\frac{d\sigma}{d\Omega_{q_2}}=\frac{1}{16}\frac{1}{(2\pi)^5}\frac{M_N^2}{s}\frac{|\mathbf{q}_2|}{|\mathbf{q}_1|}\frac{|\mathbf{q}^*|}{M^*}\times\sum\left[f(\rho\pi\pi)(q_1+q_2)_\mu\frac{\delta_{\mu\nu}+K_\mu K_\nu/m_\rho^2}{t-m_\rho^2}\times\frac{f(\rho NN)}{e}\bar{u}(P_2)M_\nu u(P_1)\right]^2 d\Omega_{q^*}dM^{*2}, \quad (5)$$

which represents the "two-body" reaction $\pi^++P\rightarrow\pi^++(P\pi^0)$ with an integration over the "internal" $P\pi^0$ phase space.

Note that thus far we have made no assumptions about the system resonating, or the reaction proceeding through an intermediate isobaric state, although for convenience we may call the $P\pi^0$ system the "isobar." Equation (5) in principle applies to any final state which can be thought of as resulting from vector boson exchange. In practice, of course, we must confine ourselves to situations where strong final-state interactions (or the absence of strong competing mechanisms) will allow this mechanism to predominate. For an indication of an alternative formulation actually using an isobaric intermediary, see Appendix II [where the (3,3) resonance is treated covariantly as a spin-$\frac{3}{2}$ "particle"]. The assumption that the system is resonant comes in when we give M_μ the form known for photoproduction at resonance.

Now the assumption of magnetic dipole implies that $M_\mu\rightarrow\propto[(1+\gamma_4)/2][3(\mathbf{q}\times\mathbf{K})-\boldsymbol{\sigma}\cdot\mathbf{q}\,\boldsymbol{\sigma}\times\mathbf{K}]$ in the $P\pi^0$ rest frame near resonance. (This gives $\mathbf{M}\cdot\boldsymbol{\varepsilon}=3\mathbf{q}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}\,\boldsymbol{\sigma}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}$, which is the $M1$ matrix element for photoproduction.) If we then evaluate the quantity in brackets in Eq. (5) in the $P\pi^0$ rest frame we get

$$\left(\frac{f(\rho\pi\pi)f(\rho NN)}{e}\right)^2\left(\frac{1}{t-m_\rho^2}\right)^2\times\sum|\bar{u}(P_2)\mathbf{M}\cdot(\mathbf{q}_1+\mathbf{q}_2)u(P_1)|^2{}_{P\pi^0\text{ rest frame}}$$

(the extra term in the propagator does not come in because $\mathbf{K}\times\mathbf{K}=0$). Observe now that the last parenthesis here is exactly what enters into the photoproduction cross section [Eq. (3)], except that where $\mathbf{K}\times\boldsymbol{\varepsilon}$ must have appeared in photoproduction $\mathbf{K}^*\times(\mathbf{q}_1^*+\mathbf{q}_2^*)=(\mathbf{q}_1^*-\mathbf{q}_2^*)\times(\mathbf{q}_1^*+\mathbf{q}_2^*)$ now appears. We therefore relate the matrix element we need to the one appearing in photoproduction:

$$\sum|\bar{u}(P_2)\mathbf{M}\cdot(\mathbf{q}_1^*+\mathbf{q}_2^*)u(P_1)|^2=[4(\mathbf{q}_1^*\times\mathbf{q}_2^*)^2/K_\gamma^2]\times\sum|\bar{u}(P_1)\mathbf{M}\cdot\boldsymbol{\varepsilon}_\gamma u(P_1)|^2,$$

where K_γ is the momentum of the photon in photoproduction of the relevant isobar mass and $\mathbf{q}_1^*$ and $\mathbf{q}_2^*$ are the momenta of the initial and final π's as seen in $P\pi^0$ rest frame. Finally, carrying out the integration over the directions of $\mathbf{q}^*$ in the $P\pi^0$ rest frame, we express Eq. (5) in terms of the photoproduction cross section:

$$\frac{d\sigma(\pi^+P\rightarrow\pi^+P\pi^0)}{d\Omega_{q_2}}=\frac{dM^*}{\pi^2}\sigma(\gamma P\rightarrow P\pi^0)\frac{|\mathbf{q}_2|}{|\mathbf{q}_1|}\frac{M^{*2}}{s}\times\frac{f^2(\rho NN)}{4\pi}\frac{f^2(\rho\pi\pi)}{4\pi}\frac{4\pi}{e^2}\left(\frac{1}{t-m_\rho^2}\right)^2\frac{(\mathbf{q}_1^*\times\mathbf{q}_2^*)^2}{K_\gamma}. \quad (6)$$

Due to the fact that the Lorentz transformation from the isobar c.m. to the over-all c.m. is along $\mathbf{q}_2$ we can simplify

$$(\mathbf{q}_1\times\mathbf{q}_2)^{*2}=\mathbf{q}_1^2\mathbf{q}_2^2(s^{1/2}/M^*)\sin^2\theta_{\text{c.m.}},$$

using the production angle in the over-all c.m., $\theta_{\text{c.m.}}$. Furthermore, we introduce the Clebsch–Gordan coefficients (C.G.) which will enter when using isotopic spin or symmetry relations to connect other reactions to the $\pi^++P\rightarrow\pi^++P+\pi^0$ via ρ^0 exchange reaction which we have used as a basis. In addition we provide for a form factor $F(t)$ which may be necessary, and set the ρ coupling constants equal,[5] giving the final formula entirely in terms of over-all c.m. quantities:

$$\frac{d\sigma}{d(\cos\theta_{\text{c.m.}})}=dM^*\frac{2}{\pi}(\text{C.G.})\frac{f_\rho^2}{4\pi}\left(\frac{f_\rho}{e}\right)^2\frac{|\mathbf{q}_1||\mathbf{q}_2|^3\sin^2\theta_{\text{c.m.}}}{(t-m_v^2)^2}\times|F(t)|^2\frac{\sigma(\gamma P\rightarrow P\pi^0)}{K_\gamma}, \quad (7)$$

where m_v is the mass of the vector boson exchanged. Values of (C.G.) for a number of reactions are given in Table I. The factors for the reactions with K particles come from assuming coupling of the ρ to the isotopic spin current [Eq. (1)], which gives $f^2(\rho^0K^-K^+)=\frac{1}{4}f^2(\rho\pi\pi)$, $2f(\rho^+K^-K^0)=f^2(\rho\pi\pi)$. Although in principle Eq. (7) holds for reactions like $K^-+P\rightarrow Y^*+\pi$ if we group (ρ,K^*) and (N^*,Y^*) into supermultiplets, it must be said that the observed differences in location and width between N^* and Y^* make the connection with $(\gamma P\rightarrow P\pi^0)$ somewhat tenuous. In such cases the reasonable thing to do would be to use the phenomenological Y^* parameters to give the shape of the resonance and then use the symmetry relations to normalize the magnitude of the cross section at one value of M^*.

It is interesting to note that at low total energies where the variation of the propagator denominator may be neglected, Eq. (7) corresponds to essentially p-wave (in the final-state) production of the isobar. The $\mathbf{q}_2^3$ dependence on the final momentum characteristic of p wave gives a rapid rise in cross section. On the other hand, at very high energies, where the t in the denominator predominates over m_v^2 the increase will be much slower. To get some idea of this behavior we make the (unjustified) assumption of neglecting $|F(t)|^2$, integrate over angles, and obtain (using again our $\pi^++P\rightarrow$

TABLE I. Coefficients (C.G.) to be used in Eq. (7) for various reactions.

Reactions	(C.G.)
$\pi^+ + P \to N^{*++} + \pi^0$	9/4
$\pi^+ + P \to N^{*+} + \pi^+$	
$\searrow P\pi^0$	1
$\searrow N\pi^+$	1/2
$\pi^- + P \to N^{*+} + \pi^-$	
$\searrow P\pi^0$	1
$\searrow N\pi^+$	1/2
$\pi^- + P \to N^{*0} + \pi^0$	
$\searrow N\pi^0$	1/2
$\searrow P\pi^-$	1/4
$\pi^- + P \nrightarrow N^{*-} + \pi^+$	
$K^+ + P \to N^{*++} + K^0$	9/4×1/2
$K^+ + P \to N^{*+} + K^+$	
$\searrow P\pi^0$	1×1/4
$\searrow N\pi^+$	1/2×1/4

$\pi^+ + P + \pi^0$ example)

$$\frac{d\sigma}{dM^*} = (\text{const})\frac{|\mathbf{q}_2|}{|\mathbf{q}_1|}\left[2A\ln\left(\frac{A+1}{A-1}\right)-4\right],$$

$$A \equiv \frac{m_{\pi^+}{}^2 - q_{10}q_{20} - m_\rho{}^2/2}{|\mathbf{q}_1|\,|\mathbf{q}_2|}. \qquad (8)$$

In the high-energy limit $m_{\pi^+}{}^2/\mathbf{q}_{1,2}{}^2 \to 0$, $A \to -1$ in such a way that the cross section diverges logarithmically:

$$d\sigma/dM^* = (\text{const})2\ln(4q^2/m_\rho{}^2). \qquad (9)$$

Of course the use of a form factor cutting down high-momentum transfer, as seems to be required by experiment, will reduce the value of the total cross section. The divergence at high energy, however, results from the peaking at low-momentum transfer, so that the problem of the removal of this divergence is tied in with the more subtle problem of very high-energy behavior.

In any event, perhaps the more interesting question is whether the estimate of the cross section makes any sense quantitatively. If we evaluate $d\sigma/dM^*$ at the resonance peak $M^*=1238$ MeV using $f_\rho{}^2/4\pi=2.2$, we get for the (const) in Eqs. (8) and (9)

$$(\text{const}) = \frac{1}{4}\frac{2}{\pi}\frac{f_\rho{}^2}{4\pi}\left(\frac{f_\rho}{e}\right)^2\frac{\sigma(\gamma P\to P\pi^0)}{K_\gamma}(\text{C.G.})$$

$$\simeq (0.11)(\text{C.G.})\ \text{mb/MeV}. \qquad (10)$$

Thus, if we were to integrate over the width of the isobar (≈ 100 MeV), we should get something in millibarns, depending on the other factors in $d\sigma/dM^*$, of course. This roughly is what is found experimentally, so at least we are around the right order of magnitude. This is perhaps not too surprising since we know from photoproduction theories[16] that the photoproduction matrix element is $e\times$(strong interaction part). Therefore in dividing by e as we have done we might expect to recover a reasonable strong interaction matrix element on general grounds. Specifically, the matrix element from photoproduction most relevant here [Ref. 16, Eq. (9)], corresponds to absorption of the photon by the nucleon anomalous magnetic moment, $\mu_P - \mu_N$. In dividing this by e, we might say (as Sakurai suggests), that we have an effect induced by "strong magnetism." In the next section we present a more detailed comparison with experiment.

III. EXPERIMENTS ON ISOBAR PRODUCTION

The N^* (and Y^*) have long been seen in the mass plots of high-energy reactions. Models, particularly for inelastic πN scattering, have been constructed which attempt to explain the total final-state spectra in terms of production through the (3,3) isobar. A recent refinement by Olsson and Yodh[17] for π^+P reactions below 1 BeV, taking into account interference between different ways of making the final state, the p-wave decay of the isobar, and assuming s-wave production of the isobar, seems to give good agreement with the final-state correlations found in experiments. Here we make no attempt to explain the entire final-state spectrum. Rather, we would like to find clear-cut cases of isobar production and discuss their features. Our discussion is more specific since we discuss not only final-state correlations, but also correlations with the incident beam, e.g., the distribution in $\theta_{\text{c.m.}}$. There are three experimental correlations in $p_{3/2}$ isobar production by mesons that we address ourselves to in particular:

(A) *The decay of the isobar in its center of mass.* We expect $(1+3(\hat{q}\cdot\hat{n})^2)d\Omega$, where $\mathbf{n}$ is the normal to the production plane, $\mathbf{n}=\mathbf{q}_1\times\mathbf{q}_2$. This corresponds to a distribution in the Treiman-Yang angle ϕ (see Sec. IV) of $(1-\frac{2}{3}\cos^2\phi)d\phi$, and a distribution in the Adair angle a of $(1-\frac{3}{5}\cos^2 a)d\Omega$. As pointed out previously,[13] this would explain why the Adair method has generally not worked on these isobars. The reason why the Adair test fails despite the complete generality used on its derivation is that while the Adair test requires that we use events at essentially 0°, the $\sin^2\theta$ in Eq. (7) causes the number of events per solid angle there to go to zero—hence no contradiction.

(B) *The distribution in the over-all center of mass with respect to* $\theta_{\text{c.m.}}$. We expect $\sin^2\theta_{\text{c.m.}}|F(t)|^2/(t-m_v{}^2)^2 d\Omega$ where for small-momentum transfers we hope $F(t)$ does

[16] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1579 (1956).
[17] M. Olsson and G. B. Yodh, Phys. Rev. Letters **10**, 353 (1963).

not vary greatly. At low energies where large momentum transfers are not attained, the $\sin^2\theta$ factor can be predominant. At high energies we expect the characteristic backward peaking of the isobar, but the differential cross section still should fall to zero for very small angles.

(C) *The distribution of events with respect to the mass of the isobar, $M^{*2} = -(q+P_2)^2$, i.e., the shape and location of the resonance.* A shift comes about in Eq. (7) through the dependence of $\sigma(\gamma P \to P\pi^0)/K_\gamma$ on M^* and the fact that $\mathbf{q}_2^3$ is a function of M^*. Alternatively, one may treat this by the phenomenological isobar method of Bergia *et al.*[18] as Kehoe[19] does, but if one takes a curve for the photoproduction cross section,[20] the difference in the two methods is minor across the resonance. We, of course, prefer the relation through photoproduction since we then have a handle on the absolute magnitude of the cross section. This shift was very nicely shown by Kehoe[19] where the resonance peak is shifted from the usual $M^* = 1238$ MeV as seen in elastic scattering to about 1200 or 1190 MeV. With respect to this $\mathbf{q}_2^3$ effect, one power of $\mathbf{q}_2$ is to be expected from phase space; the other two powers are a particular consequence of the mechanism. This leads to a concave approach to zero at the high end of the mass distribution as opposed to the square-root behavior expected from simple phase space (in the absence of beam spread). In addition to these detailed predictions, the absence of doubly charged vector mesons simply forbids certain reactions, for instance,

$$\pi^- + P \nrightarrow N^{*-} + \pi^+, \tag{11a}$$

$$K^- + P \nrightarrow Y^{*-} + \pi^+, \tag{11b}$$

for these would need the exchange of two units of charge.

To date, the most impressive evidence for the model comes from the reaction

$$K^+ + P \to N^{*++} + K^0.$$

Kehoe[19] at 910 MeV/c, which is a little above N^* threshold, using essentially all his events (i.e., making no selection for isobar mass), has found excellent agreement with points (A), (B), (C) above. In particular, the production angle distribution fits well without the use of a form factor. However, only momentum transfers up to about $17m_\pi^2$ are tested here. For the same reaction at higher energies, Crennell[21] (1.45 BeV/c) and Goldhaber[22] (1.96 BeV/c) selecting events within the N^*

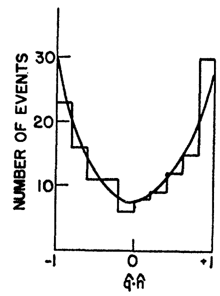


FIG. 3. Decay distribution of N^* with respect to normal to production plane in $K^+ + P \to N^{*++} + K^0$ at 1.45 BeV/c (Crennell). The solid line is the theoretical curve $1+3(\hat{q}\cdot\hat{n})^2$.

peaks, find the $1\times 3(\hat{q}\cdot\hat{n})^2$ isobar decay distribution, although the statistics are rougher and the situation may be complicated by K^* production. In Figs. 3 and 4 we reproduce Crennell's plots of the distributions with respect to the normal and the Trieman–Yang angle, respectively. Figure 5 shows the production angular distribution, curve (1) being the shape predicted without form factor dependence. Curve (2) results from using a form factor $F(t) = \exp(t/100m_\pi^2)$. The production angular distribution in t given by Goldhaber (Fig. 6 of the Ohio Conference talks[22]) shows the need for a form factor rather clearly. The curve seems to drop steeply at very forward angles as required, but the strong damping at high-momentum transfer (which reaches $110m_\pi^2$) and the forward shift of the maximum require form factor dependence. $F(t) = \exp(t/55m_\pi^2)$ for instance, gives a reasonable fit. The data at these higher energies also show a down-shift in the location of the N^* peak. To compare the total cross section with Kehoe's results at 910 MeV/c, we evaluate $d\sigma^{\mathrm{tot}}/dM^*$ at the maximum of the mass distribution ($M^* = 1200$ MeV) to get $d\sigma/dM^* = 9/8 \times 2.8 \times 10^{-3}$ mb/MeV and then integrate the mass distribution with this absolute normalization to get $\sigma^{\mathrm{tot}} \simeq 0.33$ mb Kehoe[19] finds $\sigma^{\mathrm{tot}}(K^+P \to K^0 P\pi^+) = 1.98 \pm 0.20$ mb.

This discrepancy is somewhat puzzling because the higher energy data of Crennell and Goldhaber seem to give values of σ closer to our theoretical predictions (see below).

At higher energies, where the form factor makes our simple integral Eq. (8) over production angle no longer applicable, the fairest thing to do in testing the estimate of the cross section would be to test near forward angles [even better would be

$$\mathrm{Lim}_{\cos\theta\to 1}(1-\cos^2\theta)^{-1}(d\sigma/dM^* d\Omega)],$$

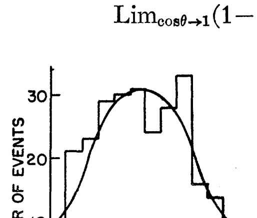


FIG. 4. Decay distribution of N^* with respect to Treiman-Yang angle in $K^+ + P \to N^{*++} + K^0$ at 1.45 BeV/c (Crennell). The solid line is the theoretical curve $1-2/3(\cos\phi)^2$.

[18] S. Bergia, F. Bonsignori and A. Stanghellini, Nuovo Cimento **16**, 1073 (1963).

[19] B. Kehoe, Phys. Rev. Letters **11**, 93 (1963); see also the results of E. Boldt, J. Duboc, N. H. Duong, P. Eberhard, R. George *et al.*, Phys. Rev. **133**, B220 (1964).

[20] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. **4**, 219 (1954).

[21] D. J. Crennell, Ph.D. thesis, Oxford University, 1963 (unpublished); and (private communication).

[22] S. Goldhaber, W. Chinowsky, G. Goldhaber, and T. O'Halloran, Bull. Am. Phys. Soc. **8**, 20 (1963). S. Goldhaber, talk at the Conference on Fundamental Particle Resonances, Ohio University, Athens, Ohio (to be published); and (private communication).

where form factor effects should be small. The Crennell data gives a value of $d\sigma/dM^*$ at $M^*=1238$ MeV of about 0.023 ± 0.002 mb/MeV while the theoretical value with neglect of the form factor is 0.036 mb/MeV. The form factor, although it does not change the shape of the angular distribution curve drastically, has a considerable effect on the total cross section, introducing here a factor approximately (0.6), resulting in 0.022 mb/MeV. This reduction is essentially a result of trying to fit the angular distribution with a monotonically decreasing form factor set to one at $t=0$. This effect is even greater in the case of the Goldhaber data, where the strongly varying form factor necessary results in a factor of roughly (0.3) in the total cross section. This gives a value of $\sigma\simeq2.4$ mb as compared with the rough value of 3 mb suggested by Goldhaber. Thus the use of the form factor in these cases seems necessary for a reasonably quantitative agreement with experimental angular distributions and cross sections. It is encouraging

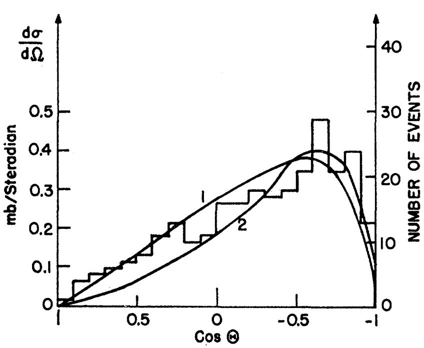


FIG. 5. Production angular distribution of the N^* in $K^++P\rightarrow N^{*++}+K^0$ at 1.45 BeV/c (Crennell). Curve (1) is the theoretical distribution with no form factor dependence. Curve (2) is the distribution with a form factor $F(t)=\exp(t/100m_\pi^2)$. The curves are drawn to arbitrary and different normalizations.

that the form factors used here to adjust the shape of the angular distribution also seem to reduce the cross section more or less correctly, although it should be emphasized that the result will depend somewhat on the particular form factor assumed.

We should also note that $\sigma\propto f_\rho{}^4$ and that f_ρ is not precisely known at present, so that we can change our theoretical value of σ by altering this factor. Alternatively, we might turn our analysis of the data around to estimate f_ρ—or more precisely $2f^2(\rho^+K^-K^0)/4\pi$, which we have taken equal to $f^2(\rho\pi\pi)/4\pi$ in virtue of assuming the ρ's universal coupling to the isospin current. If we consider our estimates with the form factors taken into account, then for the Crennell figure we need no change and for the Goldhaber estimate we can get agreement with $f^2(\rho\pi\pi)/4\pi\simeq2.5$. The present latitude on this figure is about 2.0–2.5.[5] On the other hand, were we to try to reduce the coupling constant sufficiently to make the cross sections agree without accounting for the form factors, our estimates would fall below 2.0.

Now turning to $K^-+P\rightarrow Y^*+\pi$ interactions, we find that at 1.2 BeV/c,[23] and at lower energies[24] the "wrong" resonance (11B), Y^{*-} is produced as much or more than Y^{*+} and the distribution of Y^* decay with respect to the normal is much flatter than $1+3(\hat{q}\cdot\hat{n})^2$. At 1.5 BeV/$c$,[23] Y^{*+} is somewhat favored, and by 2.2 BeV/c,[25] Y^{*-} has disappeared and the expected decay distribution is seen. Form factor dependence is again needed to get sufficient forward peaking for the π.

In $\pi+P\rightarrow Y^*+K$ reactions, Coffin *et al.*[26] at 1.5 BeV/c studied the Y^* decay distribution with respect to the normal for $\pi^-+P\rightarrow Y^{*0}+K^0$ and concluded there was no significant deviation from isotropy. At the higher momentum of 2.2 BeV/c, however, for $\pi^++P\rightarrow Y^{*+}+K^+$. Yamamoto[27] reports a distribution in agreement with $1+3(\hat{q}\cdot\hat{n})^2$ and the characteristic forward peaking requiring some form factor for the production distribution.

In πN reactions below 1 BeV, where extensive data exists on π production, the situation is complicated by the fact that over much of the range the bands on the Dalitz plot corresponding to two different isobars leading to a given final state (e.g., $\pi^++P\rightarrow\pi^++P+\pi^0$ via $N^{*++}+\pi^0$ or $N^{*+}+\pi^+$) have substantial overlap. In fact, account of this interference is important in bringing the isobaric model into accord with the data.[17] Of course, one can symmetrize the amplitudes to treat this,[28] but there is not much point in doing so unless we have some confidence that the basic mechanism is operative. At higher energies where one can escape the overlap, ρ production becomes important, but one might to hope to find some in-between region which is suitable. To this point, Tautest and Willman[29] have analyzed 1800 $\pi^++P\rightarrow N^{*++}+\pi^0$ events at 1.3 BeV/c. Although they find the backward peaking for the isobar, their N^* decay distribution with respect to the normal, $\simeq[1+0.75(\hat{q}\cdot\hat{n})^2]$ is too flat. This failing could be connected with the π^++P bump at 1.5 BeV/c or it may be, as in the $K^-+P\rightarrow Y^*+\pi$ case, that higher energies are necessary for vector meson exchange to clearly predominate.

In π^-P reactions, in addition to the above complications, there are π^-P resonances which may decay into $N^*\pi$. At low energies below the π^-P resonances where

[23] J. Button-Shafer *et al.*, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), pp. 303, 307.

[24] R. H. Dalitz, *Strange Particles and Strong Interactions* (Tata Institute of Fundamental Research, Oxford University Press, 1962), p. 97 ff.

[25] L. Bertanza, V. Brisson, P. L. Connolly *et al.*, Phys. Rev. Letters **10**, 176 (1963).

[26] C. T. Coffin *et al.*, *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 327.

[27] S. S. Yamamoto (private communication). Similar results in $\pi^++P\rightarrow Y^*+K^+$ at 2.08 BeV/c have recently been reported by H. W. J. Foelsche and H. L. Kraybill, Yale University (unpublished).

[28] R. H. Dalitz and D. H. Miller, Phys. Rev. Letters **6**, 562 (1961).

[29] G. W. Tautfest and R. B. Willman (private communication).

we might hope to get away from this problem, the extensive work of the Berkeley group[30] in this area indicates production of the "wrong" isobar (11a) favored. On the other hand, at 3.3 BeV/c, the mass plots of Guiragossian[31] show the excitation of the "right" resonance ($\pi^-+P\to N^{*+}+\pi^-$) for the final state most favored by isotopic spin, but no evidence of (11A) again supporting the idea of developing ρ exchange dominance at high energies.

In this light, it might appear as something of a puzzle that the model agrees so well with Kehoe's experiment, which is essentially at threshold for N^* production. The explanation may be found in the fact that there are no KN isobars. For in $K^-+P\to\Lambda+\pi+\pi$ or $\pi+P\to\pi+P+\pi$ the "wrong" meson can in principle form an isobar as well as the "right" one, while in $K^++P\to\pi^++P+K^0$ the absence of a K^0P isobar means that whatever is causing the formation of the isobar with the "wrong" meson is inoperative and we only see the isobars that are formed the "right" way. Furthermore, the absence of KN resonances means that there are no strong intermediate states like $Y_0^*(1815)$ or $N_{3/2}^*(1900)$ which can decay directly into an isobar and a meson as there are in K^-P or πP reactions. The decay of such states can be expected to give ratios for the charge states of our resonances as ratios of small integers and production angular distributions much more isotropic than that given by peripheral collisions. Finally, we should mention that although we have confined our discussion to $p_{3/2}$ isobars, it also is possible that vector meson exchange effects can be seen in the production of isobars of other types. For instance study of the production of a $d_{3/2}$ isobar such as in $K^-+P\to Y_0^*(1520)+K^0$ might be interesting since at least the simplest matrix element here would predict a decay distribution with respect to $\hat{n}$ for the Y_0^* curving the opposite way from the $1+3(\hat{q}\cdot\hat{n})^2$ we have found for the $P_{3/2}$ isobars. In the next section we discuss the effects of vector-meson exchange for the production of a general isobar.

IV. ISOBAR DECAY DISTRIBUTIONS

To discuss the decay distribution of an isobar of arbitrary spin and parity created by vector meson exchange (in the isobar c.m.) we analyze the process $\rho+N\to N+\pi$ in terms of the multipole expansion. (We use these particles generically; we could equally well mean $\bar{K}^*+N\to\Lambda+\pi$.) We can proceed in a manner similar to that for photoproduction, except that the polarization of the ρ may have longitudinal components.[32] The matrix element for photoproduction may be written[33]

$$M=gI+\mathbf{h}\cdot\boldsymbol{\sigma}, \tag{12}$$

where the vector $\mathbf{h}$ has three elements. To allow for longitudinal polarization we add two more parts to $\mathbf{h}$.[11] Thus we now have

$$\begin{aligned} g&=a\hat{q}\cdot\hat{K}\times\hat{\epsilon},\\ \mathbf{h}&=b\hat{\epsilon}+c(\hat{q}\cdot\hat{\epsilon})\hat{K}+d(\hat{q}\cdot\hat{\epsilon})\hat{q}+e\hat{K}+f\hat{q}, \end{aligned} \tag{13}$$

where in the isobar c.m. $\hat{q}$ is the direction of the outgoing π, $\hat{K}$ is the direction of the incoming ρ, namely, $(\mathbf{q}_1-\mathbf{q}_2)/|\mathbf{q}_1-\mathbf{q}_2|$, and $\hat{\epsilon}$ is the direction of that part of $\boldsymbol{\epsilon}$ which is perpendicular to $\mathbf{K}$. By $\boldsymbol{\epsilon}$, we now mean the spatial components, in the isobar frame, of whatever we have dotted into the current operator connecting the nucleon and the isobar. Thus, after contracting the indices in the propagator with the vertex factor V_μ for the mesons, the matrix element is essentially

$$V_\mu\frac{\delta_{\mu\nu}-K_\mu K_\nu/m_v^2}{t-m_v^2}M_\nu=\frac{\mathcal{E}_\nu M_\nu}{t-m_v^2}. \tag{14}$$

Thus in the reactions discussed so far $\boldsymbol{\epsilon}=\mathbf{q}_1+\mathbf{q}_2$; for a more complicated situation see the Appendix on ω production or the work in electroproduction.

We note that b, c, d result from electric and magnetic multipoles and e and f from longitudinal multipoles.

For clarity, we bring together the expressions for these quantities to conform to our notation[33,11]:

$$\begin{aligned}
a&=+\sum_{l=1}^{\infty}[(l+1)M_{l^+}+lM_{l^-}]P_l'(z),\\
b&=\sum_{l=1}^{\infty}[M_{l+}-M_{l-}][l(l+1)P_l(z)-zP_l'(z)]\\
&\qquad+[E_{(l-1)+}+E_{(l+1)-}]P_l'(z),\\
c&=\sum_{l=1}^{\infty}-[M_{l+}-M_{l-}][P_l'(z)+zP_l''(z)]\\
&\qquad+[E_{(l-1)+}+E_{(l+1)-}]P_l''(z),\\
d&=\sum_{l=1}^{\infty}[M_{l+}-M_{l-}-E_{l+}-E_{l-}]P_l''(z),\\
e&=\sum_{l=1}^{\infty}[(l+1)L_{l+}P_{l+1}'(z)-lL_{l-}P_{l-1}'(z)],\\
f&=\sum_{l=1}^{\infty}[lL_{l-}-(l+1)L_{l+}]P_l'(z),
\end{aligned}$$

where l refers to the orbital angular momentum of the final π and $+$ or $-$ refers to whether the total angular momentum is $l\pm\frac{1}{2}$, and $z=\hat{q}\cdot\hat{K}$. Writing the amplitude

[30] J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. **130**, 2481 (1963). This paper contains references to other work.

[31] G. T. Guiragossián, Phys. Rev. Letters **11**, 85 (1963).

[32] Note that though the ρ field has four components the subsidiary condition $K\cdot\epsilon=0$ leaves only three independent. There might appear to be some difficulty when the "peripheral" mesons to which the vector meson is coupled have unequal masses such as the πKK^* vertex; however, note that taking $\mathcal{E}$ as defined by (14) we get $K_\mu[\mathcal{E}_\mu/(K^2+m_{K^*}^2)]=\pm(m_K^2-m_\pi^2)/m_{K^*}^2$ so we still have only three independent components. Footnote 4 of Ref. 13 contains an erroneous sign on this point. In the cases where we have applied the model so far, however, the assumption of an $M1$ matrix element has been used and enters only through $\mathbf{K}\times\boldsymbol{\epsilon}$ so that any complication due to longitudinal (parallel to $\mathbf{K}$) components of $\boldsymbol{\epsilon}$ do not come up anyway.

[33] G. T. Hoff, Phys. Rev. **122**, 665 (1961).

in this way, with $\mathbf{K}$, $\boldsymbol{\varepsilon}$ represented as unit vectors, we incorporate factors coming from the magnitude of these vectors implicitly in the M, E, and L. This form is useful for calculating angular distributions and could be directly used for calculating real as well as virtual processed of the type $\rho+N \to N+\pi$. The angular distribution of the "decay" π in the isobar center of mass can now be written

$$\begin{aligned}\frac{dD}{d\Omega}&=\tfrac{1}{2}\,\mathrm{Tr}(M^{+}M)=|g|^2+\mathbf{h}\cdot\mathbf{h}^*\\&=|a|^2(\hat{q}\cdot\hat{K}\times\hat{\epsilon})^2+|b|^2+[|c|^2+|d|^2+(b^*d+bd^*)\\&\quad+z(c^*d+cd^*)](\hat{q}\cdot\hat{\epsilon})^2+|e|^2+|f|^2+z(e^*f+ef^*)\\&\quad+[(b^*f+bf^*)+(c^*e+ce^*)+z(c^*f+cf^*)\\&\qquad+z(d^*e+de^*)+(df^*+df^*)](\hat{q}\cdot\hat{\epsilon}).\qquad(15)\end{aligned}$$

Using this, we give the angular distributions for some isobars of interest, including the effects of interference between the three multipoles which contribute to each isobar of definite spin and parity.

For a $p_{3/2}$ isobar we can have $M1$, $E2$, and $L2$ contributions:

$$\begin{aligned}dD/d\Omega=&\,|M|^2[1+3(\hat{q}\cdot\hat{n})^2]+9|E|^2[1-(\hat{q}\cdot\hat{n})^2]\\&+4|L|^2(1+3z^2)+3(M^*E+ME^*)[z^2-(\hat{q}\cdot\hat{\epsilon})^2]\\&-6(M^*L+ML^*)z(\hat{q}\cdot\hat{\epsilon})+6(E^*L+EL^*)z(\hat{q}\cdot\hat{\epsilon})\,,\\&\mathbf{n}\equiv\mathbf{K}\times\boldsymbol{\varepsilon},\quad z\equiv\hat{q}\cdot\hat{K}.\end{aligned}$$

For a $d_{3/2}$ isobar we can have $E1$, $L1$, and $M2$:

$$\begin{aligned}dD/d\Omega=&\,|E|^2[1+3(\hat{q}\cdot\hat{\epsilon})^2]+9|M|^2[1-(\hat{q}\cdot\hat{\epsilon})^2]\\&+4|L|^2(1+3z^2)+3(M^*E+ME^*)[(\hat{q}\cdot\hat{n})^2-z^2]\\&-6z(\hat{q}\cdot\hat{\epsilon})[(E^*L+EL^*)+(M^*L+ML^*)].\end{aligned}$$

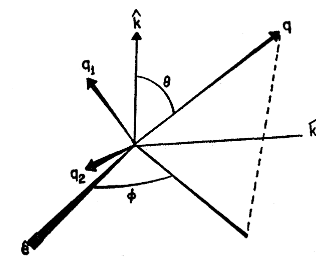
And for a $d_{5/2}$ isobar we can have $M2$, $E3$, and $L3$:

$$\begin{aligned}dD/d\Omega=&\,9|M|^2[1+5z^2-5z^4-(5z^2+1)(\hat{q}\cdot\hat{\epsilon})^2]\\&+9|E|^2[(1-10z^2+25z^4)/4+(10z^2+2)(\hat{q}\cdot\hat{\epsilon})^2]\\&+9|L|^2\tfrac{9}{2}(5z^4-6z^2+1)+\tfrac{9}{2}(M^*E+ME^*)\\&\times[1-7z^2+10z^4-(5z^2+1)(\hat{q}\cdot\hat{\epsilon})^2]\\&+9(M^*L+ML^*)3(z-5z^3)\\&+9(E^*L+EL^*)(15/2)(-z+5z^3).\end{aligned}$$

It should be remembered that generally E, M, and L are functions of t, so that their relative proportions may vary with production angle. The description may be facilitated by the introduction of a special coordinate system in the isobar c.m. (Fig. 6). Let the plane determined by $\mathbf{q}_1$ and $\mathbf{q}_2$ be the $x-z$ plane so that $\hat{K}=(\mathbf{q}_1-\mathbf{q}_2)/|\mathbf{q}_1-\mathbf{q}_2|$ is the z axis and the $\hat{e}$, the direction of $\mathbf{q}_1+\mathbf{q}_2$ perpendicular to $\hat{K}$, is then the X axis. We then can make $\mathbf{K}\times\mathbf{e}/|\mathbf{K}\times\mathbf{e}|$ the Y axis. If we then refer the direction of the outgoing π, $\hat{q}$, to the polar coordinates in this sytem, we see that ϕ is the Treiman-Yang[34] angle (i.e., the angle between the plane containing $\mathbf{q}_1$ and $\mathbf{q}_2$, and the plane containing $\mathbf{K}$ and $\mathbf{q}$), that $z=\cos\theta$, and $\mathbf{K}\times\mathbf{e}/|\mathbf{K}\times\mathbf{e}|=\hat{n}$ is the normal to the production plane. We might remark that here the Trieman-Yang criterion for spin-0 exchange simply amounts to saying that when $\mathbf{q}$ and $\mathbf{K}$ are the only available vectors, the distribution can only depend on $\mathbf{q}\cdot\mathbf{K}$. For spin-1 exchange, $\boldsymbol{\varepsilon}$ is also available, and general invariance requirements or inspection of Eq. (15) shows that we can have ϕ dependence resulting from $(\hat{q}\cdot\hat{n})^2$, $(\hat{q}\cdot\hat{\epsilon})^2$, and $\hat{q}\cdot\hat{\epsilon}$. Since generally in an arbitrary process where spin-1 is exchange $\boldsymbol{\varepsilon}=\alpha\hat{e}+\beta\hat{n}$, this means that at constant $\hat{q}\cdot\hat{K}$ we can have ϕ dependence as given by terms const, $\cos^2\phi$, $\cos\phi\sin\phi$, $\cos\phi$, and $\sin\phi$—the last two indicating interference with a longitudinal multipole.

It should be noted that the terms in $\sin\phi$ correspond to a pseudoscalar $(\hat{q}\cdot\hat{n})$ (i.e., more π's up than down) and therefore are correlated with a pseudoscalar involving the other particles in the reaction. Hence if our "peripheral" particles are spinless, or if in the case of particles with spin all we observe is their momentum, then as stated in Ref. 13, spin-1 exchange limits the distribution in $d\phi$ at fixed $\hat{q}\cdot\hat{K}$ to $A+B\cos\phi+C\cos^2\phi$.

FIG. 6. Coordinate system for describing isobar decay in isobar rest frame.



Furthermore, since the parity of the isobar corresponds to the behavior of the matrix element under the replacement $\mathbf{q}\to-\mathbf{q}$, production of an isobar of definite parity means that the distribution contains only terms even under this replacement. Since the $\cos\phi$ comes from $\hat{q}\cdot\hat{\epsilon}$, B must be odd and therefore disappears in the average over $\hat{q}\cdot\hat{K}$. Thus in the production of an isobar of definite parity, when the coordinates of the other particles are averaged over, and when we average over $\hat{q}\cdot\hat{K}$, spin-1 exchange restricts us to $A+C\cos^2\phi$. It should be noted however, as Eberhard has emphasized to us,[35] that in the case of the production of a P-wave isobar, the restriction on the ϕ distribution follows automatically, regardless of mechanism. This may be seen by going to the N^* rest frame. There, out of the five-momentum vectors in the problem only three are independent, momentum conservation and the center-of-mass condition $\mathbf{P}_2+\mathbf{q}=0$ removing two. If we now examine the distribution in $\mathbf{q}$ after $\boldsymbol{\sigma}$ has been averaged, we can take it to depend on scalar products of $\mathbf{q}$, $\mathbf{q}_1$, and $\mathbf{q}_2$ or alternatively on $\mathbf{q}$, $\mathbf{K}$, and $\boldsymbol{\varepsilon}$ $(\mathbf{K}=\mathbf{q}_1-\mathbf{q}_2,\ \boldsymbol{\varepsilon}=\mathbf{q}_1+\mathbf{q}_2)$. Now we use the fact that we have a p-wave isobar by

[34] S. B. Treiman and C. N. Yang, Phys. Rev. Letters 8, 140 (1962).

[35] Ph. Eberhard (private communication).

requiring that $\mathbf{q}$ appear linearly in the matrix element and quadratically in the distribution. Since the most complicated scalar objects we can form are scalar products and triple scalar products, we need only consider $(\mathbf{q}\cdot\mathbf{K})^2$, $(\mathbf{q}\cdot\boldsymbol{\varepsilon})^2$, $(\mathbf{q}\cdot\mathbf{n})^2$, and $(\mathbf{q}\cdot\boldsymbol{\varepsilon})(\mathbf{q}\cdot\mathbf{K})$, terms linear in $\mathbf{q}\cdot\mathbf{n}$ are excluded since it is a pseudoscalar. Since $\hat{q}\cdot\hat{\varepsilon}=\sin\theta\cos\phi$ and $\hat{q}\cdot\hat{n}=\sin\theta\sin\phi$ we obtain the restriction described above. For a general isobar, however, the restriction follows from spin-1 exchange. In fact, the restriction in general may be derived in a manner completely parallel to the argument just given by using the requirement characteristic of spin-1 exchange that $\boldsymbol{\varepsilon}$ appear linearly in the matrix element.

V. CONCLUSIONS

We have used the "ρ-photon" analogy in the context of a one-vector meson exchange model of isobar production.

The "ρ-photon" analogy is first used to select the $M1\rightarrow P_{3/2}$ transition for the $\rho+N\rightarrow N+\pi$ or $\bar{K}^*+N\rightarrow\Lambda+\pi$ vertices involved (as opposed to combinations involving the possible $E2$ or $L2$ transitions). This assumption implies that we describe the excitation of the isobar in its center of mass by the matrix element for $M1$ photoproduction ($3\mathbf{q}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}\,\boldsymbol{\sigma}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}$) with the substitutions $\mathbf{K}\rightarrow\mathbf{q}_1-\mathbf{q}_2$, $\boldsymbol{\varepsilon}\rightarrow\mathbf{q}_1+\mathbf{q}_2$, resulting in a decay distribution for the isobar $1+3(\hat{q}\cdot\hat{N})^2$ in its center of mass, a downshift in the location of the isobar peak, an over-all production angular distribution for the isobar $(\sin^2\theta/(t-m^2)^2)|F(t)|^2$, where the *ad hoc* form factor $F(t)$ is necessary at higher energies to give sufficient backward peaking for the isobar.

We have further used the quantitative aspect of the ρ-photon analogy, which relates the magnitude of a matrix element for ρ's to that for photons, to give an estimate of the cross section for isobar production via one-ρ exchange in terms of the cross section for ordinary photoproduction [Eq. (7)]. If $J_\mu{}^{\gamma v}$ and $J_\mu{}^\rho$ are the currents to which the isovector photon and the ρ^0 couple, respectively, then the quantitative statement used is that

$$(1/e)\langle J_\mu{}^{\gamma v}\rangle=(1/f_\rho)\langle J_\mu{}^\rho\rangle \quad \text{at} \quad t=0.$$

The decay distributions for the isobar predicted by the model have been seen in $\pi^++P\rightarrow Y^{*+}+K^+$ at 2.2 BeV/c, in $K^-+P\rightarrow Y^{*+}+\pi^-$ at 2.2 BeV/c, and in $K^++P\rightarrow N^{*++}+K^0$ at 910 MeV/c, 1.45 BeV/c, and 1.96 BeV/c. Although the qualitative shape of the production angular distribution is as expected in these cases, form factor dependence is needed to give sufficient backward peaking for the isobar, except in $K^++P\rightarrow N^{*++}+K^0$ at 910 MeV/c where the detailed study by Kehoe has shown agreement with all distributions predicted by the model without form factor. Yet remaining to be examined experimentally are reactions involving N^* production by π (such as $\pi^++P\rightarrow N^{*++}+\pi^0$, $\pi^-+P\rightarrow N^{*+}+\pi^0$) at a few BeV/$c$. In all the reactions mentioned, except $K^++P\rightarrow N^{*++}+K^0$, there is evidence, from angular distributions and the occurrence of "forbidden" reactions [Eq. (11)], of isobar formation at low energies in ways other than through the mechanism. It appears there may be a trend towards predominance of the mechanism at high energies, although more evidence is necessary to draw a firm conclusion on this point.

The estimate of the size of the cross section is reasonable so far as rough order of magnitude is concerned and we have made a quantitative comparison with the cross sections found at the three energies mentioned for $K^++P\rightarrow N^{*++}+K^0$. The value of $f^2(\rho^+K^-K^0)$ needed here is found by using $2f^2(\rho^+K^-K^0)=f^2(\rho\pi\pi)$ which follows from assuming universal coupling of the ρ to the isotopic current, and $f^2(\rho\pi\pi)$ is found from the experimental width for $\rho\rightarrow2\pi$. At 910 MeV/c for $K^++P\rightarrow N^{*++}+K^0$ the theoretical value for the total cross section is too small by a factor of six, while at 1.45 and 1.96 BeV/c we can fit the experimental value (if the effect of the form factor in reducing the theoretical value of the cross section is taken into account) by using $f^2(\rho\pi\pi)/4\pi=2.0$–2.5. This is in agreement with other determinations of this coupling constant. A more precise test of the calculation of the size of the cross section would be possible using a large number of experimental points at small angles so as to minimize form factor effects.

More comparison with experiment is necessary to test the model and investigate its fine points at various energies and in different reactions. The fact that the model works, however, in a variety of cases is encouraging for grouping ρ and K^*, and N^* and Y^* in common supermultiplets, and certainly lends credence to the ρ-photon analogy and those viewpoints which would assign an important role to conserved (or almost-conserved) currents in strong interactions.

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APPENDIX I. PRODUCTION OF ω^0 WITH N^*

There are indications from the work of the Columbia–Rutgers group[36] that the reaction $\pi^++P\rightarrow\omega^0+P+\pi^+$ proceeds largely through $\pi^++P\rightarrow\omega^0+N^{*++}$. Figure 3 of Ref. 36, showing the angular distributions for η, ω and ρ production, exhibits the forward peaking for ρ and ω we might expect from peripheral production.

[36] C. Alff, D. Berley, and D. Colley, Phys. Rev. Letters **9**, 322 (1962).

This peaking for the ρ production, which can proceed via one-π exchange, is sharper than that for the ω production, which cannot go this way if we assume the reaction goes through N^{*++}, and would, therefore, be suggestive of exchange of a higher mass. (The η angular distribution being flat is compatible with this reasoning since a π cannot interact with any of the presently known mesons to give an η, hence forbidding simple peripheral production.) This situation for ω production is then indicative of ρ exchange, which is the simplest possibility for $\pi^+ + P \to N^{*++} + \omega^0$ so it is worth seeing what our treatment of the ρNN^* vertex gives. The matrix element is still represented by Fig. 1, but q_2 represents the four-momentum of the ω^0 which now also possesses a polarization ϵ^ω. The introduction of the polarization of the ω makes the problem considerably less simple than when q_2 represents a spinless particle. In particular we are unable to find a simple preferred axis (which does not vary with production angle) for the decay of the isobar as we have in isobar production by spinless bosons.

Something simple can be said, however, about the ω decay. We take the effective form of the $\pi\rho\omega$ vertex to be $[f(\rho\pi\omega)/m_\pi]\epsilon_{\alpha\beta\gamma\delta}q_\alpha{}^\omega \epsilon_\beta{}^\omega K_\gamma{}^\rho \epsilon_\delta{}^\rho$ so that again the $K_\mu K_\nu/m_\rho{}^2$ term in the propagator drops out and our over-all matrix element is essentially (using $K=q_1-q_2$)

$$1/(t-m_\rho{}^2)\epsilon_{\alpha\beta\gamma\delta}q_\alpha{}^{(2)}\epsilon_\beta q_\gamma{}^{(1)}M_\delta\,, \qquad \text{(A1)}$$

where ϵ_β is the four-polarization vector of the ω and M_δ is our four current operator for the ρNN^* vertex.

Observe that in the ω rest frame, where $q_\alpha{}^{(2)} \to q_4{}^{(2)}$, the matrix element vanishes when the ω polarization is parallel to the direction of the incoming π regardless of what M_δ is or what it stands for. Now take the simple matrix element[5] for the decay $\omega \to \pi^+ + \pi^- + \pi^0$, $\epsilon_{\mu\nu\lambda\sigma}\epsilon_\mu{}^\omega K_\nu{}^{(1)}K_\lambda{}^{(2)}K_\sigma{}^{(3)}$, which by going to the ω rest frame, using $q^\omega = K^{(1)}+K^{(2)}+K^{(3)}$, is essentially $\boldsymbol{\varepsilon}^\omega\cdot(\mathbf{K}^{(1)}\times\mathbf{K}^{(2)})$. The absence of ω's with polarization parallel to $\mathbf{q}_1$ (the momentum of the incoming π in the ω rest frame) then means that the decay plane of the π's in $\omega \to \pi^+ + \pi^0 + \pi^-$ cannot be perpendicular to $\mathbf{q}_1$ and "prefers" to contain $\mathbf{q}_1$. Explicitly, if $\hat{n}_\omega$ is the normal to the decay plane, then we expect the orientation of the plane to be distributed as $1-(\hat{q}_1\cdot\hat{n}_\omega)^2$. This is only a consequence of ω production by ρ exchange, and has to do with isobar production only insofar as assuming an isobar is produced permits us to ignore one-π exchange and applies in fact equally well to $\pi+N \to N+\omega$ and similar processes via spin-1 $(-)$ exchange. The reason for this is quite general, and is similar to the argument of Smith *et al.*[37] concerning K^* production and decay. Let us consider our peripheral reaction $\pi+\rho \to \omega$ in the ω center of mass. Since both ρ and ω are 1^- the intrinsic parity of the π forces us to have $l=1$, and as in real scattering, the orbital wave function is $Y_{1,0}$ if we take the z axis in the direction of the incoming π. It can then be verified that the combination of this with the $s=1$ of the ρ cannot lead to a $J=1$ state with $J_z=0$ for the ω, so the ω can be in $J_z=\pm1$ states but not in a $J_z=0$ state. Now since the $T=0$ state of the π's in $\omega\to 3\pi$ requires the space function of the π's to be completely antisymmetric, we can represent the state of the π's by terms like $(\mathbf{K}^{(1)}\times\mathbf{K}^{(2)})_i$ times a scalar function. Note, however, that $(\mathbf{K}^{(1)}\times\mathbf{K}^{(2)})_{x,y}$ vanish when the plane of the π's becomes perpendicular to z while $(\mathbf{K}^{(1)}\times\mathbf{K}^{(2)})_z$ cannot be present since it corresponds to $J_z=0$, i.e., its form is unchanged by rotations about the z axis. This then should offer a simple check for ρ exchange with N^* production.

To treat the decay of the isobar we go into the N^* rest frame, where as before, we assume M_δ reduces to $\mathbf{M}$, the vector matrix element appropriate for $M1 \to P_{3/2}$. We now construct polarization vectors for the ω which are orthonormal and satisfy the subsidiary condition on the ω field $q_2\cdot\epsilon^\omega=0$. Let $\epsilon^{(3)}$ be the polarization vector along the ω momentum. In the ω rest frame $\epsilon^{(3)}$ should reduce to a unit vector antiparallel to the N^* momentum. Then the four-vector

$$\epsilon_\mu{}^{(3)} = [Q_\mu - (q_2\cdot Q)(q_{2\mu}/m_\omega{}^2)]/|\mathbf{Q}(\omega)|\,, \qquad \text{(A2)}$$

where Q is the four-momentum of the N^* system and $\mathbf{Q}(\omega)$ means the three-momentum of the N^* as seen in the ω rest frame, manifestly obeys $q_2\cdot\epsilon=0$, and has the desired property. This form is useful because the second term in (A2) drops out by antisymmetry when placed in (A1), and $Q_\mu \to Q_4$ in the N^* frame. Having now constructed the correct polarization vector along q_2 as seen in the isobar, we need only make $\epsilon^{(1)}$ and $\epsilon^{(2)}$ unit three-vectors (with no four component) which are perpendicular to q_2. In terms of the special coordinate system discussed in Sec. IV, we make $\epsilon^{(2)}$ along $\hat{n}=\mathbf{K}\times\mathbf{e}/|\mathbf{K}\times\mathbf{e}|$, which means that $\epsilon^{(1)}$ lies in the $\mathbf{q}_1$, $\mathbf{q}_2$ plane perpendicular to $\mathbf{q}_2$. To now evaluate

$$\sum_{i=1}^{3}|\epsilon_{\alpha\beta\gamma}\delta q_\alpha{}^{(2)}\epsilon_\beta{}^{(i)}q_\gamma{}^{(1)}M^\delta|^2$$

we use

$$\mathbf{x}^{(i)} = \mathbf{K}\times[\boldsymbol{\varepsilon}^{(i)}\times(q_4{}^{(2)}\mathbf{q}^{(1)} - q_4{}^{(1)}\mathbf{q}^{(2)})].$$

Note $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are along $\hat{n}$ and $\hat{e}$, respectively. Thus the distribution of the isobar decay (at fixed t) is

$$dD/d\Omega = |\mathbf{x}^{(1)}|^2[1+3(\hat{q}\cdot\hat{n})^2] + |\mathbf{x}^{(2)}|^2[1+3(\hat{q}\cdot\hat{e})^2] + [(M^*/|\mathbf{Q}(\omega)|)|\mathbf{K}|(\mathbf{q}_1\times\mathbf{q}_2)]^2[1+3(\hat{q}\cdot\hat{\epsilon})^2]\,, \qquad \text{(A3)}$$

where all quantities are to be evaluated in the N^* frame. This is obviously rather complicated, and detailed experimental check would involve averaging $dD/d\Omega$ over some range of t, which then brings in the propagator and form factor weighting. However, there are some less specific simple features. If we integrate over ϕ then the distribution in θ is $(1-\frac{3}{5}\cos^2\theta)d(\cos\theta)$.

[37] G. A. Smith, J. Schwartz, and D. H. Miller, Phys. Rev. Letters **10**, 138 (1963).

If we average over θ (and the internal ω coordinates) the distribution in the Treiman–Yang angle ϕ is restricted to the form $1+a\cos^2\phi$ by the spin of the isobar alone as mentioned in Sec. IV.

As for the total cross section, we take Eq. (6) and observe that

$$(\mathrm{C.G.})=9/4,\quad |\mathbf{x}^{(1)}|^2+|\mathbf{x}^{(2)}|^2+\left[\frac{M^*}{|\mathbf{Q}(\omega)|}|\mathbf{K}|(\mathbf{q}_1\times\mathbf{q}_2)\right]^2$$

must replace $(2\mathbf{q}^{(1)}\times\mathbf{q}^{(2)})^2$ and that $f(\rho\pi\pi)$ is replaced by $f(\rho\pi\omega)/m_\pi$.

APPENDIX II. COVARIANT TREATMENT OF N^*

Although there is nothing noncovariant in what we have done above, the matrix element we have used for $\rho+N\to N+\pi$ looks somewhat specialized since the explicit form we have always used refers only to the N^* rest frame. Furthermore, although in practice we have confined ourselves to situations where the final $N\pi$ system is resonant, we have not actually introduced the isobar as a particle. Therefore it may be interesting to treat the N^* or Y^* as a spin-$\frac{3}{2}$ particle and see if we can find a simple looking interaction which corresponds to the $M1$ transition we have used above. We, therefore, envision our process as $\rho+N\to N^*\to N+\pi$ and introduce the Rarita–Schwinger[38] spin-$\frac{3}{2}$ field ϕ_μ, a "four-vector" whose "components" are ordinary Dirac spinors. Since what follows can equally well apply to photoproduction, we use A_μ to represent the vector field. The N^* field has been studied elsewhere,[39] the main point for us here being that in the rest frame of the N^* the numerator of the N^* propagator

$$(M-i\gamma\cdot P)[\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu+(i/3M)(\gamma_\mu P_\nu-\gamma_\nu P_\mu)+2/(3M^2)P_\mu P_\nu]$$

reduces to $[(1+\gamma_4)/2](3\delta_{ij}-\sigma_i\sigma_j)$ with no four-components. This, when combined with the vertex $\bar{\phi}_\mu\psi(\partial_\mu\phi_\pi)$ for $N^*\to N+\pi$ leads to ($\mathbf{q}$ being the momentum of the decay π) $(3q_j-\boldsymbol{\sigma}\cdot\mathbf{q}\sigma_j)(1+\gamma_4)/2$, the $p_{3/2}$ projection operator, representing the propagation and decay of the isobar. We then need only concern ourselves with the $\rho+N\to N^*$ vertex, where the index of ϕ_μ at this vertex will go with the free index on the projection operator.

If we wish, we can construct a simple gauge-invariant interaction which always (even off resonance) gives pure $M1$ in photoproduction or for $\rho+N\to N+\pi$:

$$\frac{G}{m_\pi M^*}(\partial_\nu\bar{\phi}_\mu)\psi\epsilon_{\mu\nu\sigma\rho}K_\sigma A_\rho=\frac{G}{m_\pi}\bar{\phi}_\mu\psi\epsilon_{\mu\nu\sigma\rho}\frac{P_\nu}{M^*}K_\sigma A_\rho.$$

In the N^* rest frame $P_\nu\to P_4$ so we get $3\mathbf{q}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}\,\boldsymbol{\sigma}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}$, which is precisely the $M1$ matrix element. Thus if we give G a form factor dependence to properly reproduce the shape of the $\gamma+P\to N^{*+}\to P+\pi^0$ resonance, this interaction will give identically the results obtained before using the multipole decomposition for $M1$ in the isobar frame.[40]

On the other hand, Gourdin and Salin[41] have studied photoproduction in these terms, and have concluded that experiment requires that the photoproduction amplitude contain a small amount of $E2$. They therefore suggest using what they call $H_3=(eC_3/m_\pi)\bar{\phi}\gamma_\mu\gamma_5\psi\times(K_\mu A_\nu-K_\nu A_\mu)$, which is predominantly $M1$ with some roughly correct amount of $E2$. [If we wish to use $\epsilon_{\mu\nu\sigma\rho}$ instead of γ_5 for pseudoness, we can get the same result with $H_3'=(G/m_\pi)\bar{\phi}_\mu\gamma_\nu\psi\epsilon_{\mu\nu\sigma\rho}K_\sigma A_\rho$.] Now the $E2$ (and $L2$ for massive vector mesons) terms in the H_3's arise as recoil effects due to the motion of the proton from the small components of the spinors. This appears logical since a stationary spin-$\frac{1}{2}$ particle cannot have a quadrupole moment. For instance, the large term in H_3' is simply $(M1)$ while the small terms are

$$\frac{-K_0}{E_p+M_N}\left[1/2(M1)-1/2(E2)+1/2\frac{K^2}{K_0^2}(\mathbf{K}\cdot\boldsymbol{\varepsilon})\frac{(L2)}{\mathbf{K}^2}\right],$$

where to conform to the usage of Sec. IV (except that here $\mathbf{q}$, $\mathbf{K}$, $\boldsymbol{\varepsilon}$ are not unit vectors), we have set

$$(M1)=3\mathbf{q}\cdot\mathbf{K}\times\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}\,\boldsymbol{\sigma}\cdot\mathbf{K}\times\boldsymbol{\varepsilon},$$
$$(E2)=3i(\mathbf{q}\cdot\mathbf{K}\,\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}_T+\boldsymbol{\sigma}\cdot\mathbf{K}\,\mathbf{q}\cdot\boldsymbol{\varepsilon}_T),$$
$$(L2)=2i(3\mathbf{q}\cdot\mathbf{K}\,\boldsymbol{\sigma}\cdot\mathbf{K}-\boldsymbol{\sigma}\cdot\mathbf{q}\,\mathbf{K}^2),$$

where $\boldsymbol{\varepsilon}_T$ means the part of $\boldsymbol{\varepsilon}\perp$ to $\mathbf{K}$.

Therefore if we were to elect to use couplings of the H_3 form, we would get small amounts of $E2$ and $L2$ mixing. The recoil factor $[K_0/(E_p+M_N)]$ (to be evaluated in the isobar frame) which essentially gives the relative size of the small terms can be evaluated to give

$$\frac{K_0}{E_p+M_N}=\frac{M^{*2}-M_N^2-|t|}{(M^*+M_N)^2+|t|}.$$

In any case, if we use the pure $M1$ interaction or H_3, application of the ρ-photon analogy as described earlier, Gourdin and Salin's esitmate for the γNN^* coupling $eC_3=e(0.37)$ leads to

$$G_{\rho^0 PN^{*+}}=C_3 f_\rho\simeq(0.37)[(2.2)(4\pi)]^{1/2}\cong 2.$$

[38] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

[39] P. Federbush, M. Grisaru, and M. Tausner, Ann. Phys. (N. Y.) **18**, 23 (1962); S. Mandelstam *et al.*, *ibid.* **18**, 198 (1962). Of particular usefulness were the class notes for a course given by J. D. Jackson.

[40] Interactions can be constructed for pure $E2$ or $L2$ transitions also, but they must be concocted rather more artificially. The interaction for $E2$ and $L2$ can be written $\bar{\phi}_\mu\gamma_5(\gamma_\nu+(\gamma\cdot P/M^{*2})P_\nu)\psi T_{\mu\nu}$, or, as [by using the subsidiary condition on ϕ_μ for a real N^*, $(i\gamma\cdot P+M^*)\phi_\mu=0$] $\bar{\phi}_\mu\gamma_5[\gamma_\nu+i(P_\nu/M^*)]\psi T_{\mu\nu}$. For $E2$, $T_{\mu\nu}=K_\mu A_\nu+K_\nu A_\mu-(2K_\mu K_\nu/K^2)(K'\cdot A)$, and for $L2$, $T_{\mu\nu}=K_\mu K_\nu\times(K'\cdot A)$, where K' is a four-vector which reduces to the spatial components of K without any four-component in the isobar rest frame, $K_\mu'=K_\mu+(K\cdot P/M^*)P_\mu$.

[41] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 191, 309 (1963).